

Exam 2 – Review Sheet

Sections:13.8 - 15.2

*In addition to solving the following problems,
 make sure you have reviewed all the problems from class and homework's.*

1. The distance (position) function of an object in motion, t seconds, is given by: $s(t) = t^3 + 7t^2 + t - 1$. a) Find $v(5)$ b) Find $a(5)$	2. Given the function $f(x) = x^2 + 12x$, find the average rate of change as x changes from 3 to 5.
3. If $y = x^3 - 15x^2$, find: a) Intervals of increase/decrease. b) Relative/local Extrema using 1st/2nd derivative test. c) Intervals of Concavity and Inflection point(s). d) Graph the function, label all points and intervals.	4. Find y'' *do not simplify* a) $y = (x^4 + x)^{\frac{2}{3}}$ b) $y = \frac{3}{x^4} - \frac{1}{x}$ Find $y^{(6)}$: c) $y = x^4 - 3x^3 - 7x^2 - 6x + 9$
5. Differentiate the following: a) $y = e^x(x^3 + 4)$ d) $f(x) = e^{-4x}$ b) $y = x^3e^x$ e) $g(x) = e^{x^3+8x}$ c) $y = \frac{e^x}{x^2}$ f) $h(x) = e^{\sqrt{x^2+5}}$	6. Find the equation of the tangent line of $f(x) = 2e^{-x}$ at $x = 2$
7. Find $\frac{dy}{dx}$ if: a) $y = 5 \ln x$ d) $y = \ln(3x^2 + 4)$ b) $y = x^3 \ln x + 4x$ e) $y = \ln(\ln 5x)$ c) $y = \frac{\ln x}{x^2}$ f) $y = \ln\left(\frac{x^5-2}{x}\right)$	8. Life and Physical Sciences: The medication concentration C, in parts per million (ppm), of a medication in the body t hours after ingestion is given by the function $C(t) = 10t^2e^{-t}$. Find: a) The rate of change of the concentration. b) Find the maximum value of the concentration and the time at which it occurs.
9. Two of the 4 word problems we solved in class from section 14.3 - Maximizing/minimizing area/volume/profit functions.	

Solutions

1. a) $v(5) = 146$ b) $a(5) = 44$	2. a) $A(x) = 20$
3. a) $inc: (-\infty, 0) \cup (10, \infty)$ $dec: (0, 10)$ b) $rel\ max: (0, 0)$ $rel\ min: (10, -500)$ c) $cu: (5, \infty)$ $cd: (-\infty, 5)$ $Inflection\ point: (5, -250)$ d)	4. a) $y' = \frac{2}{3}(x^4 + x)^{-\frac{1}{3}} * (4x^3 + 1)$ OR $= \frac{2(4x^3+1)}{3(x^4+x)^{\frac{1}{3}}}$ $y'' = \frac{2}{3}[(x^4 + x)^{-\frac{1}{3}} (12x^2) - \frac{1}{3}(4x^3 + 1)^2 * (x^4 + x)^{-\frac{4}{3}}]$ b) $y' = -12x^{-5} + x^{-2}$ $y'' = 60x^{-6} - 2x^{-3}$ c) $y^{(6)} = 0$
5. a) $y' = 3x^2e^x + (x^3 + 4)e^x$ b) $y' = (x + 3)x^2e^x$ c) $y' = \frac{e^x(x-2)}{x^3}$ d) $f'(x) = -4e^{-4x}$ e) $g'(x) = (3x^2 + 8)e^{x^3+8x}$ f) $h'(x) = \frac{x e^{\sqrt{x^2+5}}}{\sqrt{x^2+5}}$	6. $y = -\frac{2}{e^2}x + \frac{6}{e^2}$
7. a) $y' = \frac{5}{x}$ d) $y' = \frac{6x}{3x^2+4}$ b) $y' = x^2 + 3x^2 \ln x + 4$ e) $y' = \frac{1}{x(\ln 5x)}$ c) $y' = \frac{1-2 \ln x}{x^3}$ f) $y' = \frac{4x^5+2}{x(x^5-2)}$	8. a) $C'(t) = 10t e^{-t}(-t + 2)$ b) The maximum value of the concentration is at $t = 2\ hours$ where the maximum occurs and $C(2) = 5.41\ ppm$ is the max concentration.